

Nonlocality of Dephasing in a Charge Qubit Interacting with a Quantum Point Contact Beam Collider

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We investigate the charge-detection-induced dephasing of a charge qubit interacting with an electronic beam collider composed of a quantum point contact. We report that, while the qubit is dephased by the partitioned beam of uncorrelated electrons, the interference of the qubit is fully restored when the two inputs are identically biased so that all the electrons suffer two-electron collision. This phenomenon is related to Fermi statistics and illustrates the peculiar *nonlocality of dephasing*. We also describe detection properties for the injection of entangled electron pairs.

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In a two-path interferometer with a *which-path* (WP) detector, the observation of interference and acquisition of the WP information are mutually exclusive [1, 2, 3]. It has been shown that dephasing (i.e., reduction of the interference) can be understood either as the acquisition of the WP information or as the back-action caused by the detector [3]. However, it has been argued that the back-action dephasing is not simply occurred as a result of the classical momentum kick in some cases [4, 5]. Owing to the recent advances in nanotechnology, mesoscopic devices now provide opportunities for investigating this issue. Indeed, WP detection in quantum interferometers has been achieved by using mesoscopic conductors [6, 7, 8, 9]. In these experiments, a quantum point contact (QPC) was used as a WP detector by probing the charge of a single electron at a nearby charge qubit composed of a quantum dot [6, 7, 8] or of a ballistic two-path conductor [9]. The particular setup we consider here is schematically drawn in Fig. 1: a charge qubit interacting with a QPC beam collider having four (two input and two output) electrodes. It has been well understood that the dephasing of the qubit is caused by the charge detection when uncorrelated electrons are injected from one of the source electrodes and partitioned by the QPC [10, 11, 12, 13, 14, 15, 16, 17, 18].

In this Letter, we report our investigations of the dephasing properties of the qubit when the detector electrons, injected from the two input electrodes, collide at the QPC. Interestingly, we find that the dephasing is suppressed (i.e., the interference is preserved) as a result of the two-particle collision. When the two electrons, injected from the two different inputs, collide at the QPC, Fermi statistics leads to antibunching of electrons. As a result, two electrons coming from the two input leads are transferred to the two different output leads because of the Pauli's exclusion principle manifested in two-particle interference. The antibunching of electrons makes it impossible, even in principle, to extract the information of the charge state despite the charge sensitivity of the scattering coefficients of the QPC detector. We argue that this shows the nonlocal nature of dephasing. We also dis-

cuss the case of entangled electron pairs injected from the two input electrodes. Our observations indicate that the information itself, rather than disturbance, indeed brings about a particle-like behavior of the qubit.

The system under consideration is composed of a charge qubit interacting with a QPC detector having four electrodes (Fig. 1(a)). This kind of detector can be constructed with the quantum Hall bar and split gates as schematically drawn in Fig. 1(b). We could also utilize the interference of the two output beams (dashed lines of Fig. 1(a,b)) for a phase-sensitive charge detection. Constructing interference [19] far away from the qubit does not influence dephasing of the qubit, but controls the efficiency of detection [16, 20]. The electron spin is neglected at this point of discussion. (Charge detection with spin-entangled electrons is discussed later.) The qubit, composed of two states, namely $|0\rangle$ and $|1\rangle$, may either be a quantum dot [6, 7] or be a two-path interferometer [9]. Creation (Annihilation) of an electron at each electrode x ($\in \alpha, \beta, \gamma, \delta$) is represented by the operator c_x^\dagger (c_x). The characteristics of the scattering of an electron at the QPC is accounted for by the scattering matrix

$$S_i = \begin{pmatrix} r_i & t'_i \\ t_i & r'_i \end{pmatrix}, \quad (1)$$

depending on the charge state i ($\in 0, 1$) of the qubit, which transforms the electron operators as

$$\begin{pmatrix} c_\gamma \\ c_\delta \end{pmatrix} = S_i \begin{pmatrix} c_\alpha \\ c_\beta \end{pmatrix}. \quad (2)$$

Charge detection and dephasing induced by the detection have been extensively studied previously when one of the input electrodes injects uncorrelated electrons [10, 11, 12, 13, 14, 15, 16, 17, 18]. In our setup of Fig. 1, this situation can be reproduced when one of the input electrodes is biased and all the other electrodes are grounded. First we briefly review the detector-induced dephasing in this case. When an electron is injected from input α , the wave function, $|\psi\rangle$, is composed of the individual wave functions of the qubit, $a_0|0\rangle + a_1|1\rangle$, and the

detector state, $c_\alpha^\dagger|F\rangle$. ($|F\rangle$ denotes Fermi sea of all the electrodes with energy lower than zero.) It evolves as

$$(a_0|0\rangle + a_1|1\rangle) \otimes c_\alpha^\dagger|F\rangle \rightarrow a_0|0\rangle \otimes |\chi_0\rangle + a_1|1\rangle \otimes |\chi_1\rangle, \quad (3)$$

where $|\chi_i\rangle = (r_i c_\gamma^\dagger + t_i c_\delta^\dagger)|F\rangle$ ($i = 0, 1$). This results in an evolution of the reduced density matrix ρ of the qubit, $\rho = \text{Tr}_{det}|\psi\rangle\langle\psi|$, obtained by tracing over the detector states of Eq.(3):

$$\rho_{ij} = a_i a_j^* \rightarrow a_i a_j^* \langle \chi_j | \chi_i \rangle = a_i a_j^* (r_i r_j^* + t_i t_j^*). \quad (4)$$

This leads to suppression of ρ_{ij} for $i \neq j$, which gives rise to dephasing of the qubit state upon continuous injection of detector electrons.

Now, let us consider the situation when electrons are injected from both of the input electrodes α and β so that two electrons collide at the QPC. In this case, the initial detector state, $c_\alpha^\dagger c_\beta^\dagger|F\rangle$, evolves into

$$|\chi_i\rangle = (r_i c_\gamma^\dagger + t_i c_\delta^\dagger)(t_i' c_\gamma^\dagger + r_i' c_\delta^\dagger)|F\rangle,$$

where i denotes the charge state of the qubit (being $i = 0$ or $i = 1$). Considering Fermi statistics, $\{c_x, c_y^\dagger\} = \delta_{xy}$, we find

$$|\chi_i\rangle = (r_i r_i' - t_i t_i') c_\gamma^\dagger c_\delta^\dagger |0\rangle = e^{i\theta_i} c_\gamma^\dagger c_\delta^\dagger |F\rangle, \quad (5)$$

where $\theta_i = \arg(r_i r_i') = \arg(t_i t_i') + \pi$ is the global phase of S_i . The latter equality of Eq. (5) is a result of the unitarity of S_i . As a result of two-particle interference and Fermi statistics, the detector state of Eq.(5) has only one particular possibility that each electron propagates into different output lead. This implies that the evolution of the density matrix of the qubit is given as

$$\rho_{ij} = a_i a_j^* \rightarrow a_i a_j^* \langle \chi_j | \chi_i \rangle = a_i a_j^* e^{i(\theta_i - \theta_j)}. \quad (6)$$

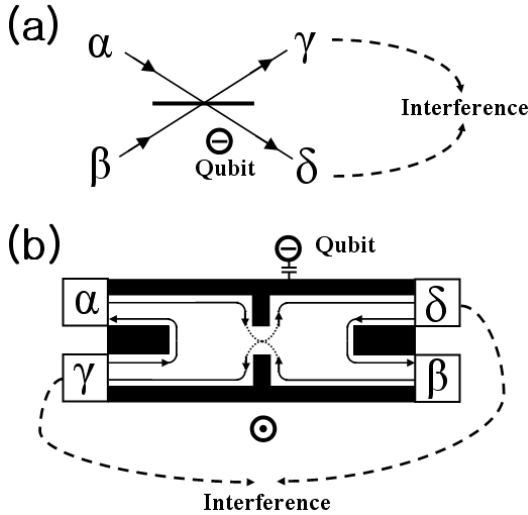


FIG. 1: (a) A schematic diagram of a charge qubit electrostatically coupled to a detector with a beam splitter and four (two input and two output) electrodes, (b) which can be realized by using the quantum Hall bar and quantum point contact.

Apparently, the two-particle collision in the detector does not reduce the interference, unlike the case of single-particle scattering.

To be specific, we consider a general case with many electrons injected from the two input electrodes α and β biased by V_α and V_β ($V_\alpha \geq V_\beta > 0$), respectively. The two output electrodes are grounded ($V_\gamma = V_\delta = 0$). The state of composite qubit-detector system initially given as

$$(a_0|0\rangle + a_1|1\rangle) \otimes \left[\prod_{0 < \varepsilon \leq eV_\beta} c_\alpha^\dagger(\varepsilon) c_\beta^\dagger(\varepsilon) \prod_{eV_\beta < \varepsilon' \leq eV_\alpha} c_\alpha^\dagger(\varepsilon') |F\rangle \right], \quad (7)$$

gets entangled upon interaction between the two subsystems. Successive events of scattering give the time dependence of the reduced density matrix as

$$\log[\rho_{ij}(t)] = \log[\rho_{ij}(0)] + \sum_\varepsilon \log \lambda_{ij}(\varepsilon), \quad (8)$$

where $\lambda_{ij}(\varepsilon)$ corresponds to the indistinguishability parameter of the detector electrons with energy ε (just as in $\langle \chi_j | \chi_i \rangle$ for the simpler cases in Eqs. (4,6)). We find

$$\lambda_{ij}(\varepsilon) = \begin{cases} e^{i(\theta_i - \theta_j)} & 0 < \varepsilon < eV_\beta \\ r_j^* r_i + t_j^* t_i & eV_\beta < \varepsilon < eV_\alpha \\ 1 & \text{otherwise} \end{cases}. \quad (9)$$

At time $t \gg h/eV_\alpha$ where the energy-time phase space is much larger than h , the summation \sum_ε can be replaced by $t \int d\varepsilon/h$. In this limit, we obtain $|\rho_{01}(t)| = |\rho_{01}(0)| \exp(-\Gamma_d t)$ with the dephasing rate Γ_d given by

$$\Gamma_d = - \int \frac{d\varepsilon}{h} \log |\lambda_{01}(\varepsilon)|, \quad (10)$$

and we get

$$\Gamma_d = - \frac{e|V_\alpha - V_\beta|}{h} \log |r_0 r_1^* + t_0 t_1^*|. \quad (11)$$

In the weak coupling limit ($r_0 r_1^* + t_0 t_1^* \sim 1$), the dephasing rate can be expanded in terms of the change in the transmission probability, $\Delta T = |t_1|^2 - |t_0|^2$, and the change in the relative scattering phase $\Delta\phi = \arg(t_1/r_1) - \arg(t_0/r_0)$. This expansion results in

$$\Gamma_d = \Gamma_T + \Gamma_\phi, \quad (12a)$$

$$\Gamma_T = \frac{e|V_\alpha - V_\beta|}{h} \frac{(\Delta T)^2}{8T(1-T)}, \quad (12b)$$

$$\Gamma_\phi = \frac{e|V_\alpha - V_\beta|}{2h} T(1-T)(\Delta\phi)^2, \quad (12c)$$

where $T = (|t_1|^2 + |t_0|^2)/2$.

Eqs.(11,12) are our central result. When only one of the input electrodes α injects electrons, that is for $V_\alpha > 0$ and $V_\beta = 0$, Eqs.(11,12) correspond to the previously

studied dephasing rate through partitioning the uncorrelated electrons [10, 11, 12, 13, 14, 15, 16, 17, 18]. Turning on the bias of the other input β results in the decrease of the dephasing rate in spite of the increasing number of detector electrons. For identical biases, $V_\alpha = V_\beta$, the dephasing rate vanishes. This intriguing result originates from the two-electron collisions which do not reduce the interference in the qubit, and can be understood as follows. As shown in Eq.(5), two electrons cannot scatter into the same output lead because of Fermi statistics. This “antibunching” makes the transport noiseless [21]. Therefore, output currents at lead γ and δ are insensitive to the charge state of the qubit (ΔT in the scattering coefficients plays no role). Furthermore, the phase sensitivity $\Delta\phi$ does not either affect the detector in any noticeable way when an interferometer is constructed between the two output leads. Therefore, *charge detection is impossible, even in principle, through the two-electron collision.*

Our result indicates the *nonlocality of dephasing*. The origin of dephasing can be interpreted either by information acquisition in the detector, or by back action of the detector causing the random fluctuation of the phase in the qubit [3]. The ‘back-action dephasing’ is often identified with “momentum kick” or local “disturbance” imposed by the uncertainty principle [1]. In the “back-action” interpretation, one might be tempted to assume a picture that the local Coulomb interaction exerts force (or a momentum kick) to the qubit leading to uncertainty of the phase. However, our result shows that this naive picture should be discarded. Injecting additional electrons at lead β does not affect the scattering matrix of Eq. (1) as long as the lead β is far apart from the qubit. If the local disturbance were the only origin of dephasing, increasing V_β would always monotonically raise the dephasing rate due to the increment of detector electrons. However, as we find above, the two-electron collision does not contribute to dephasing in spite of charge sensitivity of the scattering matrix, and it verifies the nonlocality of dephasing. We emphasize that the particle-like behavior of the qubit emerges only when the charge state information could be acquired in the detector, even if it could be done only in principle [18, 20].

Next, let us consider injection of spin-entangled electrons from the two input leads identically biased with V (Fig. 2). Some possible implementations of the spin-entangled electrons in solid-state circuits are found in Ref. 22. The “entangler” injects spin-entangled electrons to the leads α and β . The scattering matrix at the QPC is assumed to be spin-independent and is given by Eq. (1). The injected entangled triplet(singlet), prior to scattering at the QPC, is written as [23]

$$\frac{1}{\sqrt{2}}(c_{\alpha\uparrow}^\dagger c_{\beta\downarrow}^\dagger \pm c_{\alpha\downarrow}^\dagger c_{\beta\uparrow}^\dagger)|F\rangle, \quad (13)$$

where \uparrow and \downarrow represent the spin state of an electron. The

$+$ ($-$) sign in Eq (13) corresponds to the triplet(singlet) state. Upon collision at the QPC it is reduced to the qubit-charge-dependent detector state $|\chi_i^{t(s)}\rangle$ given by

$$|\chi_i^{t(s)}\rangle = \frac{1}{\sqrt{2}}\{(r_i c_{\gamma\uparrow}^\dagger + t_i c_{\delta\uparrow}^\dagger)(t'_i c_{\gamma\downarrow}^\dagger + r'_i c_{\delta\downarrow}^\dagger) \pm (r_i c_{\gamma\downarrow}^\dagger + t_i c_{\delta\downarrow}^\dagger)(t'_i c_{\gamma\uparrow}^\dagger + r'_i c_{\delta\uparrow}^\dagger)\}|F\rangle.$$

Again, Fermi statistics, $\{c_{i\sigma}, c_{j\sigma'}^\dagger\} = \delta_{ij}\delta_{\sigma\sigma'}$, is crucial in characterizing the detector properties. We find that the triplet state is simplified as

$$|\chi_i^t\rangle = \frac{1}{\sqrt{2}}e^{i\theta_i}(c_{\gamma\uparrow}^\dagger c_{\delta\downarrow}^\dagger + c_{\gamma\downarrow}^\dagger c_{\delta\uparrow}^\dagger)|F\rangle, \quad (14)$$

which leads to the indistinguishability parameter λ_{ij} of Eq. (8) as

$$\lambda_{ij}(\varepsilon) = \begin{cases} e^{i(\theta_i - \theta_j)} & 0 < \varepsilon < eV \\ 1 & \text{otherwise} \end{cases}. \quad (15)$$

As we find from Eqs. (8,15), the dephasing rate vanishes when the input electrodes inject triplet pairs just as in the collision of independent electrons. This is again due to the antibunching of the orbital wave function of electrons which provides noiseless beam upon collision. The orbital wave function of the triplet state is antisymmetric under exchange, and its statistics is equivalent to that of the independent fermions [23].

In contrast, the orbital wave function of the singlet is symmetric under two-particle exchange. Therefore we expect the detection property to be equivalent to that of bosons. Indeed, collision of the singlet pair at the QPC leads to the detector state of the form

$$|\chi_i^s\rangle = \sqrt{2}\left[r_i t'_i c_{\gamma\uparrow}^\dagger c_{\gamma\downarrow}^\dagger + t_i r'_i c_{\delta\uparrow}^\dagger c_{\delta\downarrow}^\dagger + \frac{1}{2}(t_i t'_i + r_i r'_i)(c_{\gamma\uparrow}^\dagger c_{\delta\downarrow}^\dagger + c_{\delta\uparrow}^\dagger c_{\gamma\downarrow}^\dagger)\right]|F\rangle. \quad (16)$$

This singlet detector state, unlike those of the triplet (Eq. (14)) and of the two independent electrons (Eq. (5)), has a “bunching” property, which enhances the shot noise [23]. The bunching is perfect for the symmetric partitioning at the QPC (that is $|t_i| = |r_i| = 1/\sqrt{2}$) where $t_i t'_i + r_i r'_i = e^{i\theta_i}(|r_i|^2 - |t_i|^2) = 0$. In this case, the two electrons are always found at the same lead (γ or δ). Moreover, this bunching enhances the charge sensitivity

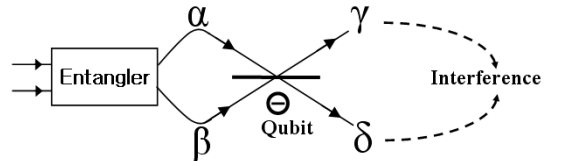


FIG. 2: A schematic diagram of a charge qubit and a detector that injects spin-entangled electrons.

of the detector. For the detector injecting singlet pairs, we find that the dephasing rate Γ_d^s is given as

$$\Gamma_d^s = -\frac{2eV}{h} \log |\lambda_{01}^s|, \quad (17)$$

where $\lambda_{01}^s = \langle \chi_1^s | \chi_0^s \rangle = 2(r_1^* t_1'^* r_0 t_0' + t_1^* r_1'^* t_0 r_0') + (t_1^* t_1'^* + r_1^* r_1'^*)(t_0 t_0' + r_0 r_0')$ is the indistinguishability factor for a singlet pair. Factor 2 in the right hand side of Eq. (17) comes from the spin degeneracy, which was not taken into account in Eq. (11). In the weak measurement limit, Γ_d^s is given by an algebraic sum of the two different contribution: $\Gamma_d^s = \Gamma_T^s + \Gamma_\phi^s$, where the current-sensitive (Γ_T^s) and the phase-sensitive (Γ_ϕ^s) contributions are given as

$$\Gamma_T^s = \frac{eV}{h} \frac{(\Delta T)^2}{T(1-T)}, \quad (18a)$$

$$\Gamma_\phi^s = \frac{eV}{h} 4T(1-T)(\Delta\phi)^2, \quad (18b)$$

The dephasing rate is now enhanced (by eight times) compared to the case with only one electrode injecting uncorrelated electrons ($V_\beta = 0, V_\alpha = V$ in Eq. (12)). Taking into account the simultaneous injection from the two inputs and the spin degeneracy, the number of injected electrons for a given time is four times larger in the case of injecting singlet states. This means that the charge sensitivity of the singlet pairs is twice as compared to that of the uncorrelated single electrons. This originates from the bunching behavior of the orbital wave function. It is noteworthy that this scheme may be utilized to achieve more precise charge detection [24].

In conclusion, we have analyzed the properties of charge detection in a QPC when the electrons from different inputs collide. We have found that the properties of dephasing are determined by the statistics of the incident electrons, and demonstrated the nonlocality of dephasing. This verifies that, while the dephasing is directly related to the which-path information in general, it cannot be simply understood in terms of local disturbance that washes out the coherence.

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- [1] R. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1965), Vol. 3.
- [2] W. K. Wootters and W. H. Zurek, Phys. Rev. D **19**, 473 (1979).
- [3] A. Stern, Y. Aharonov and Y. Imry, Phys. Rev. A **41**, 3436 (1990).
- [4] M. O. Scully, B.-G. Englert and H. Walther, Nature (London) **351**, 111 (1991).
- [5] S. Dürr, T. Nonn, and G. Rempe, Nature **395**, 33 (1998).
- [6] E. Buks, R. Schuster, M. Heiblum, D. Mahalu and V. Umansky, Nature (London) **391**, 871 (1998).
- [7] D. Sprinzak, E. Buks, M. Heiblum and H. Shtrikman, Phys. Rev. Lett. **84**, 5820 (2000).
- [8] M. Avinun-Kalish, M. Heiblum, A. Silva, D. Mahalu and V. Umansky, Phys. Rev. Lett. **92**, 156801 (2004).
- [9] I. Neder, M. Heiblum, D. Mahalu, and V. Umansky, Phys. Rev. Lett. **98**, 036803 (2007).
- [10] I. L. Aleiner, N. S. Wingreen, and Y. Meir, Phys. Rev. Lett. **79**, 3740 (1997).
- [11] S. A. Gurvitz, Phys. Rev. B **56**, 15215 (1997).
- [12] Y. Levinson, Europhys. Lett. **39**, 299, (1997).
- [13] G. Hackenbroich, B. Rosenow, and H. A. Weidenmüller, Phys. Rev. Lett. **81**, 5896 (1998).
- [14] L. Stodolsky, Phys. Lett. B **459**, 193 (1999).
- [15] M. Büttiker and A. M. Martin, Phys. Rev. B **61**, 2737 (2000).
- [16] D. V. Averin and E. V. Sukhorukov, Phys. Rev. Lett **95**, 126803 (2005).
- [17] K. Kang, Phys. Rev. Lett. **95**, 206808 (2005).
- [18] G. L. Khym and K. Kang, Phys. Rev. B **74**, 153309 (2006).
- [19] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature **422**, 415 (2003).
- [20] K. Kang, Phys. Rev. B. **75**, 125326 (2007).
- [21] R. C. Liu, B. Odom, Y. Yamamoto, and S. Tarucha, Nature **391**, 263 (1998).
- [22] G. Burkard, J. Phys.: Condens. Matter **19**, 233202 (2007).
- [23] G. Burkard, D. Loss, and E. V. Sukhorukov, Phys. Rev. B **61**, R16 303 (2000).
- [24] B. Yurke, Phys. Rev. Lett. **56**, 1515 (1986).